Stability and negative tensions in 6D brane worlds

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ABSTRACT: We investigate the dynamical stability of warped, axially symmetric compactifications in anomaly free 6D gauged supergravity. The solutions have conical defects, which we source by 3-branes placed on orbifold fixed points, and a smooth limit to the classic sphere-monopole compactification. Like for the sphere, the extra fields that are generically required by anomaly freedom are especially relevant for stability. With positive tension branes only, there is a strict stability criterion (identical to the sphere case) on the charges present under the monopole background. Thus brane world models with positive tensions can be embedded into anomaly free theories in only a few ways. Meanwhile, surprisingly, in the presence of a negative tension brane the stability criteria can be relaxed. We also describe in detail the geometries induced by negative tension codimension two branes.

Contents

1. Introduction

Much has been written about brane world solutions to chiral gauged 6D supergravity. Gibbons, Güven and Pope (GGP) [1] found a wide class of such solutions, with 4D Poincaré symmetry and axial symmetry in the transverse dimensions. Surrounding work focusing on the brane world interpretation of these backgrounds was made in [2, 3]. Other classes of solutions have also been found, including those which break the axial symmetry [4], activate the hyperscalar fields [5], have 4D de Sitter/anti de Sitter slicings [6] or time-dependent behaviour [7 – 9]. The model additionally admits string solutions having dyonic charges, with trivial [10] or active [11] hyperscalars. All these backgrounds are interesting because,

among other things, they provide a framework in which to build cosmological models (for a review see [12]).

In order for these backgrounds to be phenomenologically relevant, however, we would like them to be stable against small perturbations.[1] So far, the GGP solutions have proven to be classically marginally stable, despite the fact that they all but one (i.e. the sphere-monopole limit) break supersymmetry (see [13], and also [14 – 16]). Meanwhile, we might also want to consider the 6D theory to be a low energy approximation to some consistent theory of quantum gravity, such as string theory. If this is the case we must insist on certain consistency constraints, and in particular, since the theory is chiral, we must insist on anomaly freedom.

In general, chiral 6D supergravity suffers from a breakdown of local symmetries due to gravitational, gauge and mixed anomalies. For certain gauge groups and hypermultiplet representations these anomalies can be cancelled via a Green-Schwarz mechanism [17]. This is entirely analogous to what happens in 10D, where the anomalies cancel only for a few models, namely those with gauge groups: $SO(32)$, $E_8 \times E_8$, $E_8 \times U(1)^{248}$ and $U(1)^{496}$. In 6D the consistency constraints are weaker, and by now a number of anomaly free models have been discovered [17 – 20]. In table 1 we present three of the known examples which have a large enough gauge group to include the Standard Model of Particle Physics.

The structure of these anomaly free models seems suggestive that some of them may indeed be somehow related to critical string theory or M-theory [21]. Meanwhile, with regards to the stability of the brane world compactifications, the extra degrees of freedom required for anomaly cancellation cannot be ignored. Marginal stability was affirmed in [13], for the GGP solution in the Salam-Sezgin model, which has just an Abelian $U(1)_R$ gauge group and no hypermatter. We now ask if there is a similar dynamics in anomaly free models, which have larger field contents.

To this end, we may draw some lessons from the old literature on sphere compact-ifications, which are supported by monopole backgrounds [22]. The stability of sphere compactifications for nonsupersymmetric theories was studied in [23, 24], and for anomaly free supergravity theories in [17 – 19]. Whilst the models are stable in the presence of just a Maxwell gauge group, for Yang-Mills theories a tachyonic instability is generically found in the scalars descending from the gauge fields and charged under the $U(1)$ monopole back-ground [24]. For example, it turns out that only one of the anomaly free models presented in table 1 has a stable sphere compactification if the $U(1)$ monopole is embedded in a non-Abelian factor of the gauge group: the $E_7 \times E_6 \times U(1)_R$ model with the monopole embedded in E_6 [17, 19, 25].

Our main focus in the present work is then on the scalar perturbations of the gauge fields charged under the $U(1)$ monopole background, as possible sources of instability. We analytically solve the linearized dynamics of these fields, and in particular derive their full Kaluza-Klein mass spectra. In this way we are able to identify some conditions for stability, and we observe the previous behaviour as well as some surprises.

Our results can be summarised as follows. Conical-GGP solutions which incorporate

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[1]Or at least not to exhibit runaways that are too fast.

Gauge Group	Hyperino Representation
$E_7 \times E_6 \times \mathrm{U}(1)_R$	$(\mathbf{912}, \mathbf{1})_0$
$E_7 \times G_2 \times \mathrm{U}(1)_R$	$(\mathbf{56}, \mathbf{14})_0$
$F_4 \times \mathrm{Sp}(9) \times \mathrm{U}(1)_R$	$(\mathbf{52}, \mathbf{18})_0$

Table 1: Some examples of anomaly free models with gauge groups containing $\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)$ [17–19]. There are also many other models, including e.g. one with just an Abelian $\mathrm{U}(1)_R$ gauge group, and the anomalies cancelled by 245 neutral hypermultiplets [20].

only positive tension brane sources are stable only for very special matter contents and monopole embeddings. Specifically, the stability criteria observed in the sphere limit persists for these more general solutions. The sphere's stability criteria is also sufficient to ensure stability for conical-GGP solutions in the presence of negative tensions (placed on orbifold fixed points). However, remarkably, we find that negative tension branes can relax these conditions, and render unstable sphere compactifications stable.

Let us end with an overview of the paper. In section 2 we briefly review the theory and its warped, axially symmetric brane world solutions. We also discuss some physical aspects of the background, in particular emphasising that the geometry induced by the backreaction of negative tension branes is well-defined. Then, in section 3 we classify all the scalar perturbations present in the model and identify how the various sectors decouple. We argue that possible tachyonic instabilities should lie in the scalar fluctuations of the gauge fields orthogonal to the background monopole in the Lie algebra of the gauge group. Therefore, we turn in section 4 to the Kaluza-Klein mass spectra of these fields, and dedicate section 5 to the consequences of these spectra for the stability. We end with some conclusions, and leave for the appendices some details on the algebra.

2. 6D supergravity and its axially symmetric solutions

In order to fix our conventions, we begin by reviewing the 6D chiral gauged supergravity and brane world solutions that interest us. Then, in the following subsection, we will collect some details about the background geometry and topology which will later prove to be important.

2.1 The theory and solution

We consider 6D supergravity with a general matter content, whose gauge group $\mathcal{G}$ is a product of simple groups that include a $\mathrm{U}(1)_R$ gauged R-symmetry. For example we could take the anomaly free group $\mathcal{G} = E_7 \times E_6 \times \mathrm{U}(1)_R$, under which the hyperinos are charged as $\Psi \sim (\mathbf{912}, \mathbf{1})_0$ [17]. The bosonic action takes the form[2] [26]

$$S_B = \int d^6X \sqrt{-G} \left[\frac{1}{\kappa^2}R - \frac{1}{4}\partial_M\sigma\partial^M\sigma - \frac{1}{4}e^{\kappa\sigma/2}Tr\left(F_{MN}F^{MN}\right) \right.$$

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[2]We choose signature $(-, +, \ldots, +)$, and define $R = G^{MN}(\partial_P\Gamma^P_{MN} - \partial_M\Gamma^P_{PM} + \ldots)$. The index M runs over $0, 1, \ldots, 5$. For fermionic terms see [26].

$$-\frac{\kappa^2}{48}e^{\kappa\sigma}H_{MNP}H^{MNP} - g_{\alpha\beta}(\Phi)D_M\Phi^\alpha D^M\Phi^\beta - \frac{8}{\kappa^4}e^{-\kappa\sigma/2}v(\Phi)\bigg], \quad (2.1)$$

where κ represents the 6D Planck scale and g is the gauge coupling constant, which in fact represents a collection of independent gauge couplings including that of the $U(1)_R$ subgroup, g_1. The field σ is the dilaton, F_{MN} is the field strength of the gauge field, $\mathcal{A}_M$, and H_{MNP} is the Kalb-Ramond field strength, which contains a Chern-Simons coupling as follows:[3]

$$H_{MNP} = \partial_M B_{NP} + Tr\left[F_{MN}\mathcal{A}_P - \frac{g}{3}\mathcal{A}_M\left(\mathcal{A}_N \times \mathcal{A}_P\right)\right] + 2\ perms. \quad (2.2)$$

The metric $g_{\alpha\beta}(\Phi)$ is on the target manifold of the hyperscalars, and here the index α runs over all the hyperscalars. The dependence of the scalar potential on Φ^α is such that its minimum is at $\Phi^\alpha = 0$, where it takes a positive-definite value, $v(0) = g_1^2$, due to the R-symmetry gauging [27, 10].

We refer to [15] for the equations of motion that follow from (2.1). A general class of configurations with 4D Poincaré symmetry and axial symmetry in the transverse dimensions, is:

$$ds^2 = G_{MN}dX^M dX^N = e^{A(\rho)}\eta_{\mu\nu}dx^\mu dx^\nu + d\rho^2 + e^{B(\rho)}d\varphi^2,$$
$$\mathcal{A} = \mathcal{A}_\varphi(\rho)Q d\varphi, \qquad \sigma = \sigma(\rho),$$
$$H_{MNP} = 0, \qquad \Phi^\alpha = 0, \quad (2.3)$$

with $0 \leq \rho \leq \overline{\rho}$ and $0 \leq \varphi < 2\pi$. Here $\mu, \nu = 0, 1, 2, 3$ and Q is a generator of a $U(1)$ subgroup of a simple factor of $\mathcal{G}$, satisfying $Tr\left(Q^2\right) = 1$.

In the following we shall also use the radial coordinate defined by

$$\xi(\rho) \equiv \int_0^\rho d\rho' e^{-A(\rho')/2}, \quad (2.4)$$

whose range is $0 \leq \xi \leq \overline{\xi}$. In this frame the metric reads

$$ds^2 = e^{A(\xi)}\left(\eta_{\mu\nu}dx^\mu dx^\nu + d\xi^2\right) + e^{B(\xi)}d\varphi^2. \quad (2.5)$$

Given the above ansatz, the general solution has been found by GGP [1]. We will focus on a subset of this general solution, namely that which contains singularities no worse than conical. The explicit conical-GGP solution[4] is then [1]:

$$e^A = e^{\kappa\sigma/2} = \sqrt{\frac{f_1}{f_0}}, \quad e^B = \alpha^2 e^A \frac{r_0^2 \cot^2(\xi/r_0)}{f_1^2},$$
$$\mathcal{A} = -\frac{4\alpha}{q\kappa f_1}Q d\varphi, \quad (2.6)$$

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[3]We define the cross-product as $(\mathcal{A}_M \times \mathcal{A}_N)^{\hat{I}} = f^{\hat{I}\hat{J}\hat{K}}\mathcal{A}_M^{\hat{J}}\mathcal{A}_N^{\hat{K}}$, with $f^{\hat{I}\hat{J}\hat{K}}$ the structure constants of $\mathcal{G}$. The index $\hat{I}$ runs over the full Lie algebra of $\mathcal{G}$, and later we will use I to label those directions orthogonal to that of the $U(1)$ monopole.

[4]The coordinate ξ is related to the coordinate r in [1] by $r = r_0 \cot(\xi/r_0)$.

where q and α are generic real numbers. Also,

$$f_0 \equiv 1 + \cot^2\left(\frac{\xi}{r_0}\right), \quad f_1 \equiv 1 + \frac{r_0^2}{r_1^2}\cot^2\left(\frac{\xi}{r_0}\right), \tag{2.7}$$

with $r_0^2 \equiv \kappa^2/(2g_1^2)$, $r_1^2 \equiv 8/q^2$.

The conical-GGP configuration is, however, a solution to the equations of motion only outside the points $\xi = 0$ or $\xi = \bar{\xi} \equiv \pi r_0/2$. This is because as $\xi \to 0$ or $\xi \to \bar{\xi}$, the metric tends to that of a cone, with respective deficit angles

$$\delta = 2\pi\left(1 - |\alpha|\frac{r_1^2}{r_0^2}\right) \quad \text{and} \quad \bar{\delta} = 2\pi\left(1 - |\alpha|\right), \tag{2.8}$$

and corresponding delta-function behaviours in the Ricci scalar. Note that α appears in the deficit angles only through its modulus, since the metric (2.6) depends only on the square of α, and so is insensitive to its sign. In order to promote the solution to a global one, we introduce two 3-brane sources into the system, each with action:

$$S_b = -T \int d^4y \sqrt{-\det\left(G_{MN}\partial_\alpha Y^M \partial_\beta Y^N\right)}, \tag{2.9}$$

where $Y^M(y^\alpha)$ are the brane embedding fields, y^α are the worldvolume coordinates, $\alpha = 0, \ldots, 3$ and the tensions are respectively [28]

$$T = 2\delta/\kappa^2 \quad \text{and} \quad T = 2\bar{\delta}/\kappa^2. \tag{2.10}$$

In this way we arrive at a warped codimension-two brane world construction, in which the 3-branes can localize bulk fields [15] and/or support 4D fields in such a way as to realize the Standard Model of Particle Physics.

Finally, we note that one can obtain the "rugby ball" compactification [29] simply by setting $r_0 = r_1$. In this case the background value of the dilaton is zero, and therefore the stability analysis that we are going to present will also be applicable to the rugby ball solution of non-supersymmetric 6D Einstein-Yang-Mills models. Moreover, we can smoothly retrieve the sphere compactification by taking $r_1 \to r_0$ and $\alpha \to 1$.

2.2 Geometry and topology

We continue our discussion on the background configuration by considering in more detail its geometry. In particular, it is interesting to note that the parameters appearing in the deficit angles, α and r_1, are not fixed by the EOM but rather represent moduli. However, from (2.8), we can see that the deficit angles are both bounded from above by 2π. This becomes an upper bound on the brane tensions that can be described by the conical-GGP solution, because of the matching conditions in (2.10).

Meanwhile it is also clear that the deficit angles can take arbitrary large negative values. We emphasise here that manifolds with negative deficit angles are perfectly well defined, and can even be made at home with a piece of paper and a pair of scissors. Take for example the simplest case of a cone. A cone with positive deficit angle is obtained by
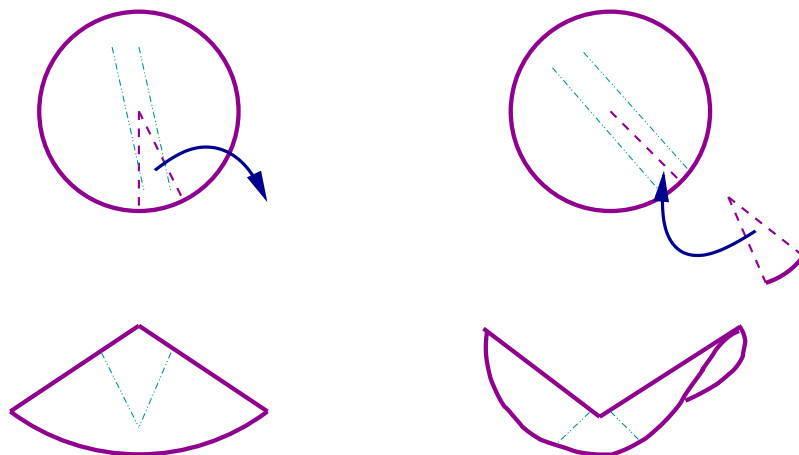
Figure 1: Construction of a cone and saddle-cone, respectively, by splicing out and in, respectively, a wedge starting from the flat disk. For the cone, lines that were parallel on the disk remain parallel until they pass on either side of the apex, when they begin to converge. For the saddle-cone they diverge once passing the apex. Notice that although the saddle-cone appears to break the axial-symmetry, this is only an effect of the embedding into 3D. A 2D being would indeed observe the axial symmetry.

splicing a wedge out of a flat disk and gluing together the edges. Similarly, a cone with negative deficit angle is obtained by splicing a wedge *into* a flat disk.[5] The result will be a manifold which is flat everywhere apart from at the apex of the cone. Lines that were parallel on the disk remain parallel until they pass on either side of the apex, after which they will begin to diverge, just as for the standard cone they would converge (see figure 1). Moreover, there is no lower bound on the deficit angle. For example, one could imagine adding $\pi/3$ wedges successively to the flat disk *ad infinitum*.

Discretized versions of manifolds with negative deficit angles can be found in the solid-state literature on Carbon Nanostructures, in which nanocones with negative disclination angles are appropriately named "saddle-cones" [31]. These provide another nice way to visualize the geometries, including those with, say, deficit angle less than -2π. Referring to figure 2, we observe that beginning with a flat planar lattice of regular hexagons, and splicing in a wedge of $\pi/3$, one ends with another lattice of hexagons, now taking the form of a (flat) saddle, and with the central hexagon replaced by a heptagon. A deficit angle of $-\pi$ would correspond to a central enneagon, one of -2π to a dodecagon, and so on.

On the other hand, we should note from eq. (2.10) that in our scenario the negative deficit angles are sourced by negative tension branes. It is well known that negative tension branes generically suffer from classical and quantum instabilities [32]. We will return to this issue in the following section.

Having understood the geometry of the model, we must also take care of its topology. The Euler number of the internal manifold is two, indicating the topology of a sphere. This

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[5]As this manuscript was being prepared ref. [30] appeared, baptizing manifolds with spherical topology and negative deficit angles as "Apple-like".
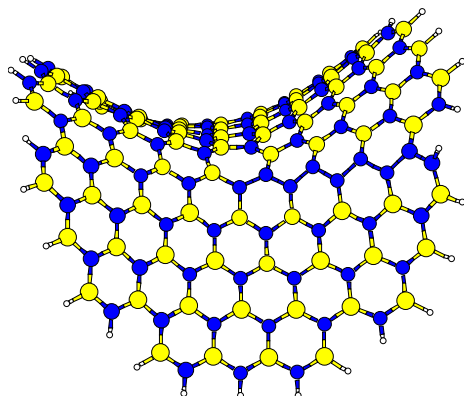
Figure 2: A nanocone with a disclination angle $-\pi/3$. Reproduced from ref. [31] by kind permission of the authors.

can be seen from direct calculation, as well as by showing that the manifold can be covered by two holomorphic coordinate patches, which are related at the intersection by $z = 1/\zeta$ (see below). We must therefore take care to ensure that the gauge field background is well-defined as $\xi \to 0$ and $\xi \to \overline{\xi}$. Indeed the expression for the gauge field background in eq. (2.6) is well-defined in the limit $\xi \to 0$, but not as $\xi \to \overline{\xi}$. We must therefore use a different patch to describe the $\xi = \overline{\xi}$ limit, and in the overlap this must be related to the patch including $\xi = 0$ by a single-valued gauge transformation. This leads to a Dirac quantization condition, which for a field interacting with $\mathcal{A}$ through a charge e gives

$$-e\,\frac{4\alpha\overline{g}}{\kappa q} = -e\,\alpha\frac{r_1}{r_0}\frac{\overline{g}}{g_1} = N\,, \tag{2.11}$$

where N is an integer that is called monopole number and $\overline{g}$ is the gauge coupling constant corresponding to the background gauge field. For example, if $\mathcal{A}$ lies in $\mathrm{U}(1)_R$, then $\overline{g} = g_1$. Generally different fields have different charges e^i, which correspond to several monopole numbers N^i; as we shall see this aspect is important in the stability analysis of the conical-GGP solutions. The charges e^i can be computed once we have selected the gauge group, since they are eigenvalues of the generator Q.

As has long and often been noted, the Dirac quantization leads to a constraint, relating the tensions of the two 3-branes which can be described by the conical-GGP solution and the bulk gauge couplings [1]. For example, embedding the monopole in the $\mathrm{U}(1)_R$ gauge sector requires the presence of at least one negative tension brane [1, 3].

3. The scalar fluctuations

We now consider scalar fluctuations about the brane world solution, with the aim of studying its stability. The bulk perturbations which are scalars from the brane point of view can be written as:

$$\left\{\delta G^\mu_\mu, \delta G_{\rho\rho}, \delta G_{\varphi\varphi}, \delta G_{\rho\varphi}, \delta\sigma, \delta\zeta, \delta B_{\rho\varphi}, \delta\mathcal{A}_\rho, \delta\mathcal{A}_\varphi, \delta\Phi\right\}\,. \tag{3.1}$$

Here, $\delta\zeta$ is the four-dimensional dual of the fluctuation in the Kalb-Ramond field $\delta B_{\mu\nu}$. Moreover, the presence of branes introduces additional dynamical degrees of freedom, which are the brane-bending modes corresponding to fluctuations in their position in six-dimensional spacetime:

$$\left\{\delta Y^M\right\} . \tag{3.2}$$

Let us now discuss these various fluctuations in turn.

3.1 Brane bending sector

As we discussed in the introduction, the main focus of the present paper is the study of the fluctuations in a sub-sector of the 6D gauge fields which gives rise to instabilities in the spherical limit. As we shall see, at the bilinear level this sub-sector does not mix with any of the other fields, in particular it does not mix with the fluctuations in the position of the branes (3.2). In principle we could concentrate on the relevant gauge sector right away. However, since we shall also consider negative tension branes we will show that we can modify our internal space by dividing it by a well defined action of a global discrete group and thereby eliminating some of the potentially harmful modes. It should be stressed, however, that our analysis of the fluctuations and the solution of the rather complex eigenvalue problem will not be affected by this orbifolding. Neither our conclusion concerning the stability of the aforementioned gauge sub-sector will be affected.

By choosing the so-called static gauge for the worldvolume coordinates, $\partial_\alpha Y^\mu = \delta_\alpha^\mu$, we see that each brane carries two physical fields which correspond to their bending in the transverse dimensions:

$$\{\delta Y^\rho(x), \delta Y^\varphi(x)\} . \tag{3.3}$$

We call the 4D fields in (3.3) brane bending modes.[6] Notice that these fields are four dimensional, and so do not lead to a Kaluza-Klein tower.

In general we can expect the brane bending modes to mix with the bulk metric fluctuations at the bilinear level. Moreover, it is well known that negative tension branes can lead to energies unbounded from below and instability. However, we can choose to place the branes at orbifold fixed points in such a way that they are not free to fluctuate. An orbifold symmetry can indeed project out some physical modes, like the brane bending modes, because it consists of the additional physical condition that the action is invariant under the orbifold transformations. At the same time, we observe that negative tension branes do not necessarily lead to instabilities even in the presence of some brane bending modes, as shown for example in [35, 36]. Therefore, introducing an orbifold that, as we shall see, project out the brane bending modes might not be necessary to have stability, even in the presence of a negative tension brane.

To find an orbifold projection that serves our purpose, we need a global description of the background internal manifold, and in particular one in which both the background

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[6]We observe that this terminology is not always adopted in the literature: the author of [33] calls the modes in (3.3) brane coordinate fields, whereas the authors of [34] choose the name branons.

brane positions can be well described. To this end, we cover the manifold with two coordinate patches. First we define the complex coordinates $z, \bar{z}$, with:

$$z = \eta \, e^{i\varphi} \qquad \text{where} \quad \eta = e^{\int_{\rho_0}^{\rho} e^{-B/2}} \tag{3.4}$$

for some arbitrary ρ_0. The metric on the internal manifold becomes:

$$ds^2 = d\rho^2 + e^B d\varphi^2 = \frac{e^{B(\rho(z\bar{z}))}}{z\bar{z}} \, (dz \, d\bar{z}) \, . \tag{3.5}$$

Using the behaviour of e^B as $\rho \to 0$ ($e^B \to (\alpha \, r_1^2/r_0^2)^2 \rho^2$), one can show that the brane at $\rho = 0$ is now well described by the single point $z = 0$. Meanwhile, in analogy with standard stereographic coordinates, the point $\rho = \bar{\rho}$ cannot be covered by $z, \bar{z}$. We therefore use a different coordinate patch to describe the brane there, defined by $\zeta, \bar{\zeta}$ with:

$$\zeta = \frac{1}{\eta} \, e^{-i\varphi} \, . \tag{3.6}$$

In this patch the brane at $\rho = \bar{\rho}$ is well-defined at $\zeta = 0$, and it is the brane at $\rho = 0$ which is not covered. In the overlap, the two coordinate systems are related by $\zeta = 1/z$, $\bar{\zeta} = 1/\bar{z}$. Notice that this confirms that the internal space is conformally CP^1.

Now one can immediately see that the background brane positions are the fixed points under the orbifold identification

$$z \leftrightarrow -z \, , \quad \zeta \leftrightarrow -\zeta \tag{3.7}$$

Thanks to the argument that we have just discussed, the orbifold action defined in (3.7) is globally defined: this is because the two patches defined in (3.4) and (3.6) together cover the whole internal space. The brane fluctuations, $\delta Y^z(x), \delta Y^{\bar{z}}(x)$ and $\delta Y^\zeta(x), \delta Y^{\bar{\zeta}}(x)$, being both[7] odd under the orbifold action, are thus projected out. This orbifold also acts on the bulk fields in a way that the combinations of the bulk fields, which mix with the brane bending modes, are odd[8] and therefore vanish on the branes. This observation also shows that projecting out the brane bending modes with the orbifold defined here is consistent with the equations of motion.

Here we also observe that our orbifolding is different from the one considered in [35], in the context of 5D brane worlds: there the authors introduced an orbifold whose fixed point is at the perturbed brane position $\overline{Y} + \delta Y$, under which the bending mode δY is even (invariant), whereas we are considering an orbifold whose fixed point is at the background brane position $\overline{Y}$, under which δY is odd. This explains why the brane bending modes were not projected out in [35], contrary to what happens in our case.

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[7]We note that our proof that the brane bending modes of both the branes are projected out by (3.18) is made for a particular internal space having spherical topology, and therefore such a result might not be applicable in the most general codimension 2 scenario.

[8]This is a consequence of the fact that Identification (3.7) can be regarded as a particular coordinate transformation and the lagrangian is invariant under coordinate transformations: the combinations of the bulk fields that mix with the brane bending modes must be odd in order to create an even (invariant) term in the lagrangian.

3.2 Hyperscalars

It is straightforward to see that the hyperscalars do not mix with the other sectors at the level of the bilinear action. Moreover, we can quite directly conclude they cannot give rise to tachyonic instabilities: this is a consequence of the fact that the potential $v(\Phi)$ has a global minimum at $\Phi = 0$, and also that the metric $g_{\alpha\beta}(\Phi)$ is positive definite. Therefore, the contributions from the 6D potential and the 2D Laplacian to the mass-squareds in the Kaluza-Klein mass spectrum are both positive.

3.3 Scalars from the gauge fields

The scalar fluctuations descending from the gauge fields can be divided into two separate classes. First there are the fluctuations:

$$\{\delta\mathcal{A}_\rho\,Q, \delta\mathcal{A}_\varphi\,Q\}\,, \tag{3.8}$$

with the gauge group generator Q corresponding to that of the background monopole. These will be included in the discussion of the following subsection. Second, there are the fluctuations

$$\left\{\delta\mathcal{A}_\rho^I\,T^I, \delta\mathcal{A}_\varphi^I\,T^I\right\}, \tag{3.9}$$

with T^I being the generators orthogonal to Q, that is $Tr(T^I Q) = 0$. At the bilinear level the second group does not mix with the first, and nor with any of the other sectors in (3.1) and (3.2), even if we do not introduce the orbifold. This is a consequence of the form of the bulk action (2.1), of the Kalb-Ramond field strength (2.2) and of the brane action (2.9). We will therefore return to the fields (3.9), which are the main focus in the present paper, in section 4. Possible instabilities are generally lurking in this sector.

3.4 Salam-Sezgin sector

The remaining fields are those which also correspond to the minimal Salam-Sezgin model, that is with just one $U(1)_R$ gauge multiplet:

$$\left\{\delta G_\mu^\mu, \delta G_{\rho\rho}, \delta G_{\varphi\varphi}, \delta G_{\rho\varphi}, \delta\sigma, \delta\zeta, \delta B_{\rho\varphi}, \delta\mathcal{A}_\rho\,Q, \delta\mathcal{A}_\varphi\,Q\right\}\,. \tag{3.10}$$

Let us here recall the long-known result that the Salam-Sezgin sphere model is marginally stable, with two and only two massless scalar modes, and a Kaluza-Klein tower of heavy positive mass-squared modes [17, 37, 25]. One of the massless modes corresponds to the spontaneous breaking of the global classical scaling symmetry.[9] The other is guaranteed by the unbroken Kalb-Ramond gauge symmetry.[10] Since the massless modes are protected by symmetry arguments, we can argue that small deformations of the sphere solution to the conical-GGP solution must remain marginally stable.

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[9]The EOMs are invariant under the constant rescaling $G_{MN} \to \lambda\,G_{MN}$ and $e^{\kappa\sigma/2} \to \lambda\,e^{\kappa\sigma/2}$. Note that this is only a classical symmetry because the action rescales as $S_B \to \lambda^2\,S_B$.

[10]This symmetry acts as $B \to B + d\Lambda$, where B is the two-form with components B_{MN}, and Λ a one-form gauge parameter. Then the components $B_{\mu\nu}$ are dual to a massless 4D scalar. Meanwhile, the directions $B_{\rho\varphi}$ provide the Goldstone boson that is to be eaten by the massive $U(1)$ gauge field in the direction of the monopole.

Moreover, ref. [13] analysed explicitly a subsector of (3.10), namely the axially-symmetric perturbations corresponding to those members that are even under a certain parity symmetry. By imposing that the perturbations preserve the conical singularities,[11] a single massless mode was found, corresponding to the classical scaling symmetry enjoyed by the field equations.[12] All other modes were shown to have positive squared-masses.

4. Linear analysis for scalar fluctuations of the gauge fields

We will now complete the stability analysis for brane world compactifications in anomaly free models, by considering the final sector

$$\left\{ \delta\mathcal{A}_\rho^I T^I, \delta\mathcal{A}_\varphi^I T^I \right\}. \tag{4.1}$$

In fact, these fields are of particular interest. Indeed, it has long been known that this sector — and only this sector — can in general contain tachyonic modes in its Kaluza-Klein spectra for the sphere compactification with a monopole background [24]. It is therefore interesting to ask what happens to these tachyons if one considers the conical-GGP configuration, which as we have seen is a warped deformation of the sphere compactification.

As an example, we could consider the anomaly free model of $E_7 \times E_6 \times \mathrm{U}(1)_R$, with the monopole embedded in E_6. In this case, the low-energy gauge group is:[13] $E_7 \times \mathrm{SO}(10) \times \mathrm{U}(1)_R \times \mathrm{U}(1)_{KK}$, and the fluctuations covered by our analysis are two sets of scalars transforming under $E_7 \times \mathrm{SO}(10) \times \mathrm{U}(1)_R$ as $(\mathbf{133}, \mathbf{1})_0 + (\mathbf{1}, \mathbf{45})_0 + (\mathbf{1}, \mathbf{16})_0 + (\mathbf{1}, \overline{\mathbf{16}})_0 + (\mathbf{1}, \mathbf{1})_0$, and in various representations of $\mathrm{U}(1)_{KK}$ depending on the monopole number.

In order to keep the analysis of the present sector as general as possible, we choose here to keep also the modes that are projected out by the orbifold boundary conditions discussed above. Indeed, the complete spectrum arising from Sector (4.1) can be relevant in the case in which the presence of the brane bending modes does not lead to any instabilities and therefore the orbifold is not necessary. On the other hand, if the orbifold were necessary, it would only project out some of the modes in the full spectra derived below, and so would not introduce new instabilities. Indeed, as we have already established, the fields in (4.1) are completely decoupled (in the bilinear lagrangian) from the rest; in particular they do not mix with the brane bending modes (3.2). We shall explain how to impose the orbifold conditions on (4.1) after the derivation of the complete spectrum from (4.1) (see the end of Subsection 5.2).

4.1 Bilinear action

Let us then consider the bilinear action for the fluctuations

$$\begin{aligned}
V_\rho &= V_\rho^I T^I \equiv \delta\mathcal{A}_\rho^I T^I, \\
V_\varphi &= V_\varphi^I T^I \equiv \delta\mathcal{A}_\varphi^I T^I
\end{aligned} \tag{4.2}$$

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[11]Actually, the general GGP solutions with worse than conical singularities were also considered in [13].

[12]The second massless mode can be expected amongst the perturbations that are odd under the parity symmetry.

[13]In the sphere limit the $\mathrm{U}(1)_{KK}$ is promoted to $\mathrm{SU}(2)_{KK}$.

around the background (2.3). Since T^I is orthogonal to Q, the perturbed action simplifies considerably, having contributions only from the gauge kinetic term in (2.1). After fixing to the light-cone gauge (see reference [38] and appendix A for details), the result can be written as:[14]

$$S_2(V,V) = -\frac{1}{2} \int d^6X \sqrt{-\hat{G}} \, Tr \left[\partial_\mu V_i \partial^\mu V^i + D_i V_j D^i V^j - 2(\partial_r \hat{A})^2 V_r^2 \right.$$
$$\left. -2(\partial_r \hat{A}) V_r D_i V^i + \hat{R}_{ij} V^i V^j + 2\overline{g} \, F_{ij} V^i \times V^j \right], \qquad (4.3)$$

where we have introduced $dr \equiv e^{\kappa\sigma/4} d\rho$, and the indices i, j run over r, φ. Also for compactness, we have defined:

$$\hat{A} \equiv A + \phi, \qquad \hat{B} \equiv B + \phi, \qquad \phi \equiv \kappa\sigma/2$$
$$\hat{G}_{MN} dX^M dX^M \equiv e^{\hat{A}} \eta_{\mu\nu} dx^\mu dx^\nu + dr^2 + e^{\hat{B}} d\varphi^2, \qquad (4.4)$$

and all indices are raised and lowered with the background metric $\hat{G}_{MN}$. $\hat{R}_{ij}$ are the internal components of the Ricci-tensor defined from the metric $\hat{G}_{MN}$, and F_{ij} refers to the background field strength.

Moreover, recall that the covariant derivative in general includes the gauge field background. In particular, we have

$$D_\varphi V_j = \nabla_\varphi V_j - i\overline{g} \, \mathcal{A}_\varphi [Q, V_j] \qquad (4.5)$$

with ∇_φ the Lorentz covariant derivative. Below we will choose a basis of generators such that:

$$[Q, T^I] = e^I T^I, \qquad (4.6)$$

which means that in general they will not be Hermitian. However, we choose the normalization $Tr(T^{I\dagger} T^J) = \delta^{IJ}$, and also define $[T^I, T^J] = if^{IJK} T^K$. Also, e^I is the corresponding charge under the U(1) monopole. For example, for the $E_7 \times E_6 \times$U(1) model, we have $e^I \neq 0$ for the **16** and $\overline{\mathbf{16}}$. The Dirac quantization condition (2.11) then gives $-e^I 4\alpha\overline{g}/(\kappa q) = N^I$. In the following, we suppress the index I.

Finally, since our internal space is topologically S^2, we shall impose that the fluctuations are periodic functions of φ. Therefore, we can apply the following Fourier decomposition:

$$V_j(X) = \sum_m V_{jm}(x, r) e^{im\varphi} \qquad (4.7)$$

with m an integer, $-\infty < m < \infty$.

4.2 The equations of motion and boundary conditions

Next we vary the above action with respect to V_r and V_φ, perform the Fourier decompositions (4.7), and project onto the Fourier number m. After a long but standard calculation

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[14]In (4.3) the fluctuation fields V_i have been normalized in a way that the canonical factor $-1/2$ appears in front of the kinetic terms.

we eventually obtain the following coupled equations of motion:

$$e^{-\hat{A}} M_m^2 V_{rm} = -\partial_r^2 V_{rm} - \left(2\partial_r \hat{A} + \frac{1}{2}\partial_r \hat{B} \right) \partial_r V_{rm}$$
$$+ \left[e^{-\hat{B}} (m - e\overline{g}\,\mathcal{A}_\varphi)^2 - (\partial_r \hat{A})^2 - \frac{1}{2}\partial_r \hat{A}\partial_r \hat{B} - \partial_r^2 \hat{A} - \frac{1}{2}\partial_r^2 \hat{B} \right] V_{rm}$$
$$+ i e^{-\hat{B}} \left[\left(\partial_r \hat{B} - \partial_r \hat{A} \right) (m - e\overline{g}\,\mathcal{A}_\varphi) + 2e\overline{g}\,\partial_r \mathcal{A}_\varphi \right] V_{\varphi m} \tag{4.8}$$

and

$$e^{-\hat{A}} M_m^2 V_{\varphi m} = -\partial_r^2 V_{\varphi m} - \left(2\partial_r \hat{A} - \frac{\partial_r \hat{B}}{2} \right) \partial_r V_{\varphi m} + e^{-\hat{B}} (m - e\overline{g}\,\mathcal{A}_\varphi)^2 V_{\varphi m}$$
$$- i \left[\left(\partial_r \hat{B} - \partial_r \hat{A} \right) (m - e\overline{g}\,\mathcal{A}_\varphi) + 2e\overline{g}\,\partial_r \mathcal{A}_\varphi \right] V_{rm}, \tag{4.9}$$

where M_m^2 are the eigenvalues of $\eta^{\mu\nu}\partial_\mu\partial_\nu$.

At the same time, the variation leads to the following boundary conditions [39, 15]:

$$\int d^4x dr \partial_r \left[e^{2\hat{A}+\hat{B}/2} \delta V_{rm}^\dagger \left(\partial_r - (\partial_r \hat{A}) \right) V_{rm} \right] = 0 \tag{4.10}$$

and

$$\int d^4x dr \partial_r \left[e^{2\hat{A}-\hat{B}/2} \delta V_{\varphi m}^\dagger \left(\partial_r - \frac{1}{2}(\partial_r \hat{B}) \right) V_{\varphi m} \right] = 0 \tag{4.11}$$

where we used $Tr(T^{I\dagger}T^J) = \delta^{IJ}$. Eqs. (4.10) and (4.11) represent the requirement that the boundary terms in the integration by parts, which is performed in deriving the equations of motion, vanish. Since the action that we are varying is bilinear, Conditions (4.10) and (4.11) are bilinear as well. Notice that these are the weakest boundary conditions possible, which we must apply in order to achieve such a requirement. In principle, with some physical motivation, we could apply stronger boundary conditions. However, for the purposes of the stability analysis, since stronger boundary conditions would only eliminate modes from the physical spectrum, we prefer to remain as general as possible. Conditions (4.10) and (4.11) ensure that the hamiltonian of the effective Schroedinger problem associated to (4.8) and (4.9) is hermitian[15] [39, 15].

Our objective is then to solve these coupled linearized equations, together with their boundary conditions, in order to deduce the behaviour of the perturbations.

4.3 The Schroedinger problem

We proceed by transforming the system into a pair of coupled Schroedinger equations plus boundary conditions. This is achieved by introducing the coordinate ξ, defined in (2.4), and the new variables:

$$V_{1m}(x,\xi) \equiv e^{\hat{A}/4+\hat{B}/4} V_{\xi m}(x,\xi), \tag{4.12}$$
$$V_{2m}(x,\xi) \equiv e^{3\hat{A}/4-\hat{B}/4} V_{\varphi m}(x,\xi). \tag{4.13}$$

---

[15]We will explicitly show how to reduce (4.8) and (4.9) to an effective Schroedinger problem and comment more on the hermiticity of its hamiltonian in Subsection 4.3.

The equations of motion (4.8) and (4.9) then become:

$$M^2 V_1 = -V_1'' + \frac{1}{16}\left[-4\hat{A}'' - 4\hat{B}'' + \left(\hat{A}' + \hat{B}'\right)^2 + 16 e^{\hat{A}-\hat{B}}\left(m - e\overline{g}\,\mathcal{A}_\varphi\right)^2\right] V_1$$
$$+ i e^{\hat{A}/2 - \hat{B}/2}\left[\left(\hat{B}' - \hat{A}'\right)(m - e\overline{g}\,\mathcal{A}_\varphi) + 2e\overline{g}\,\mathcal{A}_\varphi'\right] V_2 \tag{4.14}$$

and

$$M^2 V_2 = -V_2'' + \frac{1}{16}\left[12\hat{A}'' - 4\hat{B}'' + \left(3\hat{A}' - \hat{B}'\right)^2 + 16 e^{\hat{A}-\hat{B}}\left(m - e\overline{g}\,\mathcal{A}_\varphi\right)^2\right] V_2$$
$$- i e^{\hat{A}/2 - \hat{B}/2}\left[\left(\hat{B}' - \hat{A}'\right)(m - e\overline{g}\,\mathcal{A}_\varphi) + 2e\overline{g}\,\mathcal{A}_\varphi'\right] V_1 \,, \tag{4.15}$$

where $' \equiv \partial_\xi$ and we have suppressed the index m. In other words, the system can then be described in the following way:

$$\begin{pmatrix} -\partial_\xi^2 + U_1(\xi) & iC(\xi) \\ -iC(\xi) & -\partial_\xi^2 + U_2(\xi) \end{pmatrix}\begin{pmatrix} V_1(x,\xi) \\ V_2(x,\xi) \end{pmatrix} = M^2 \begin{pmatrix} V_1(x,\xi) \\ V_2(x,\xi) \end{pmatrix}, \tag{4.16}$$

where the Schroedinger potentials are given by

$$U_1 \equiv \frac{1}{16}\left[-4\hat{A}'' - 4\hat{B}'' + \left(\hat{A}' + \hat{B}'\right)^2 + 16 e^{\hat{A}-\hat{B}}\left(m - e\overline{g}\,\mathcal{A}_\varphi\right)^2\right], \tag{4.17}$$

$$U_2 \equiv \frac{1}{16}\left[12\hat{A}'' - 4\hat{B}'' + \left(3\hat{A}' - \hat{B}'\right)^2 + 16 e^{\hat{A}-\hat{B}}\left(m - e\overline{g}\,\mathcal{A}_\varphi\right)^2\right], \tag{4.18}$$

and the coupling function is

$$C \equiv e^{\hat{A}/2 - \hat{B}/2}\left[\left(\hat{B}' - \hat{A}'\right)(m - e\overline{g}\,\mathcal{A}_\varphi) + 2e\overline{g}\,\mathcal{A}_\varphi'\right]. \tag{4.19}$$

At this stage we can observe that for the conical solution (2.6) the two Schroedinger potentials are degenerate:

$$U_1 = U_2 \equiv U\,. \tag{4.20}$$

It is therefore straightforward to diagonalize the system. Indeed, transforming into the basis:

$$V_\pm(x,\xi) \equiv \frac{1}{\sqrt{2}}\left(V_1(x,\xi) \pm iV_2(x,\xi)\right) \tag{4.21}$$

the matrix equation (4.16) becomes:

$$\begin{pmatrix} -\partial_\xi^2 + U(\xi) + C(\xi) & 0 \\ 0 & -\partial_\xi^2 + U(\xi) - C(\xi) \end{pmatrix}\begin{pmatrix} V_+(x,\xi) \\ V_-(x,\xi) \end{pmatrix} = M^2 \begin{pmatrix} V_+(x,\xi) \\ V_-(x,\xi) \end{pmatrix}. \tag{4.22}$$

Having decoupled the equations, we should now consider the boundary conditions in terms of the new basis (4.21). The sum and difference of (4.10), (4.11) lead to the following constraints:

$$\int d^4x\, d\xi\, \partial_\xi\left[\delta V_+^\dagger\left(\partial_\xi - \frac{1}{4}\hat{B}'\right) V_+ + \delta V_-^\dagger\left(\partial_\xi - \frac{1}{4}\hat{B}'\right) V_-\right] = 0 \tag{4.23}$$

and

$$\int d^4 x d\xi \partial_\xi \left[\delta V_+^\dagger \left(\partial_\xi - \frac{1}{4} \hat{B}' \right) V_- + \delta V_-^\dagger \left(\partial_\xi - \frac{1}{4} \hat{B}' \right) V_+ \right] = 0. \tag{4.24}$$

Here we have used that $\hat{A}' \to 0$ at the boundaries, as can be seen from (2.6) and (2.7). Notice that since the dynamics of V_+ and V_- are decoupled, we can consider both them and their variations to be independent. Therefore, choosing first $V_-(x,\xi) = 0$ and $\delta V_-(x,\xi) = 0$, and then $V_+(x,\xi) = 0$ and $\delta V_+(x,\xi) = 0$, we see that the boundary conditions can be equivalently expressed as[16]

$$\int d^4 x d\xi \partial_\xi \left[\delta V_+^\dagger \left(\partial_\xi - \frac{1}{4} \hat{B}' \right) V_+ \right] = 0 \tag{4.25}$$

and

$$\int d^4 x d\xi \partial_\xi \left[\delta V_-^\dagger \left(\partial_\xi - \frac{1}{4} \hat{B}' \right) V_- \right] = 0. \tag{4.26}$$

In fact, these conditions ensure that the Hamiltonians in the Schroedinger equations (4.22) are Hermitian [39, 15], as we discussed at the end of Subsection 4.2, and thus have real eigenvalues and orthonormal sets of eigenfunctions. We shall therefore refer to them as Hermiticity Conditions (HCs).

Finally, we recall that in order to derive the physical spectrum of the perturbations we must also impose the additional constraint of a finite kinetic term in (4.3). Expressed in term of the decoupled fluctuations (4.21), this implies that:

$$-\frac{1}{2} \int d^4 x d\xi \left[\partial_\mu V_+^\dagger \partial^\mu V_+ \right] < \infty \tag{4.27}$$

and

$$-\frac{1}{2} \int d^4 x d\xi \left[\partial_\mu V_-^\dagger \partial^\mu V_- \right] < \infty. \tag{4.28}$$

4.4 The spectrum

Having decoupled the equations in terms of the dynamical fields $V_{+m}(x,\xi)$ and $V_{-m}(x,\xi)$ (restoring momentarily the index m), let us decompose these fields in the standard Kaluza-Klein way:

$$\begin{aligned} V_{+m}(x,\xi) &= \mathcal{V}_{+m}(x)\psi_{+m}(\xi), \\ V_{-m}(x,\xi) &= \mathcal{V}_{-m}(x)\psi_{-m}(\xi). \end{aligned} \tag{4.29}$$

Now, for ψ_+ the Schroedinger potential can be written explicitly as:[17]

$$U(\xi) + C(\xi) = U_0 + u \cot^2 \left(\frac{\xi}{r_0} \right) + \overline{u} \tan^2 \left(\frac{\xi}{r_0} \right), \tag{4.30}$$

---

[16]Indeed, $(V_+(x,\xi), 0)$ and $(0, V_-(x,\xi))$ are both well-defined solutions to the two-by-two Schroedinger system (4.22). Then (4.25) and (4.26) are both necessary and sufficient boundary conditions. We also checked explicitly that (4.23), (4.24) and (4.25), (4.26) are equivalent for our final solutions.

[17]Note that here we have discarded the delta-function contributions to the potential, since they are dominated by stronger singularities [15].

where

$$r_0^2 U_0 \equiv \frac{1}{2} + 2m\omega - 2(m - N)\bar{\omega} + 2m(m - N)\omega\bar{\omega},$$

$$r_0^2 u \equiv \frac{3}{4} + m^2\omega^2 + 2m\omega, \quad r_0^2 \overline{u} \equiv \frac{3}{4} + (m - N)^2\bar{\omega}^2 - 2(m - N)\bar{\omega}, \qquad (4.31)$$

and

$$\omega \equiv (1 - \delta/2\pi)^{-1}, \qquad \bar{\omega} \equiv (1 - \overline{\delta}/2\pi)^{-1}. \qquad (4.32)$$

Moreover, the HC reduces to:

$$\lim_{\xi \to \overline{\xi}} \psi_+^* \left(-\partial_\xi + \frac{1}{2}\frac{1}{\xi - \overline{\xi}}\right)\psi_+ - \lim_{\xi \to 0} \psi_+^* \left(-\partial_\xi + \frac{1}{2\xi}\right)\psi_+ = 0 \qquad (4.33)$$

and the finiteness of the kinetic energy (4.27) becomes simply the normalizability condition (NC) on the wavefunction:

$$\int d\xi |\psi_+|^2 < \infty. \qquad (4.34)$$

The potential, HC and NC for ψ_- are identical to those for ψ_+, but replacing $m \to -m$ and $N \to -N$.

The problem is now of exactly the same form as that treated in [15], where the spectrum for gauge field and fermion fluctuations was derived. We can therefore follow the same steps made there. The Schroedinger equation can be transformed into the hypergeometric equation:

$$z(1 - z)\partial_z^2 y + [c - (a + b + 1)z]\partial_z y - aby = 0, \qquad (4.35)$$

by defining

$$z = \cos^2\left(\frac{\xi}{r_0}\right), \qquad \psi = z^\gamma (1 - z)^\beta y(z), \qquad (4.36)$$

and, for $\psi = \psi_+$,

$$\beta \equiv \frac{1}{4}(3 + 2m\omega), \; \gamma \equiv \frac{1}{4}[3 - 2(m - N)\bar{\omega}], \; c \equiv 2 - (m - N)\bar{\omega},$$

$$a \equiv \frac{1}{2}\left\{3 + m\omega - (m - N)\bar{\omega} + \sqrt{r_0^2 M^2 + 1 + [m\omega - (m - N)\bar{\omega}]^2}\right\},$$

$$b \equiv \frac{1}{2}\left\{3 + m\omega - (m - N)\bar{\omega} - \sqrt{r_0^2 M^2 + 1 + [m\omega - (m - N)\bar{\omega}]^2}\right\}. \qquad (4.37)$$

The solution can then be expressed in terms of Gauss's hypergeometric function, F. For $c \neq 1$ the two linearly independent solutions are:

$$y_1(z) \equiv F(a, b, c, z), \quad y_2(z) \equiv z^{1-c}F(a + 1 - c, b + 1 - c, 2 - c, z), \qquad (4.38)$$

and so the general solution to the Schroedinger equation is

$$\psi = K_1\psi_1 + K_2\psi_2, \qquad (4.39)$$

with

$$\psi_i \equiv z^\gamma (1 - z)^\beta y_i \qquad (4.40)$$

and $K_{1,2}$ the integration constants. For $c = 1$ we have $\psi_1 = \psi_2$ but we can construct a linearly independent solution using the Wronskian method and the general solution reads

$$\psi = K_1\psi_1 + K_2\psi_1 \int^{\xi} \frac{d\xi'}{\psi_1^2(\xi')}. \qquad (4.41)$$

We must now impose the NC (4.34) and HC (4.33) to select the physical modes. The explicit calculations are given in appendix C of [15], and so we do not repeat them here. The final result is the following.

The wavefunctions for ψ_+ are:

$$\psi_+ \propto z^{\gamma}(1 - z)^{\beta}F(a, b, c, z), \quad for \quad m \leq N + 1/\bar{\omega}, \qquad (4.42)$$
$$\psi_+ \propto z^{\gamma+1-c}(1 - z)^{\beta}F(a + 1 - c, b + 1 - c, 2 - c, z), \quad for \quad m > N + 1/\bar{\omega}. \quad (4.43)$$

The corresponding squared masses are:

- For $m \leq -1/\omega$ and $m \leq N + 1/\bar{\omega}$

$$M^2 = \frac{4}{r_0^2}\left\{n(n + 1) - \left(n + \frac{1}{2}\right)[m\omega + (m - N)\bar{\omega}] + m(m - N)\omega\bar{\omega}\right\}. \qquad (4.44)$$

- For $-1/\omega < m \leq N + 1/\bar{\omega}$

$$M^2 = \frac{4}{r_0^2}\left\{\left(n + \frac{3}{2}\right)^2 - \frac{1}{4} + \left(n + \frac{3}{2}\right)[m\omega - (m - N)\bar{\omega}]\right\}. \qquad (4.45)$$

- For $N + 1/\bar{\omega} < m \leq -1/\omega$

$$M^2 = \frac{4}{r_0^2}\left\{n(n - 1) - \left(n - \frac{1}{2}\right)[m\omega - (m - N)\bar{\omega}]\right\}. \qquad (4.46)$$

- For $m > -1/\omega$ and $m > N + 1/\bar{\omega}$

$$M^2 = \frac{4}{r_0^2}\left\{n(n + 1) + \left(n + \frac{1}{2}\right)[m\omega + (m - N)\bar{\omega}] + m(m - N)\omega\bar{\omega}\right\}. \qquad (4.47)$$

Here $n = 0, 1, 2, \ldots$. The masses given in (4.44) and (4.45) correspond to the wave function (4.42) whereas the masses given in (4.46) and (4.47) correspond to the wave function (4.43). The spectrum for ψ_- can be obtained from that above simply by transforming $m \to -m$ and $N \to -N$.

5. Stability analysis

In the preceding section we have analytically derived the complete Kaluza-Klein spectrum for the scalar fluctuations of the 6D gauge field, for directions in the Lie Algebra of $\mathcal{G}$ orthogonal to the background monopole. We are now ready to analyze the stability of these fluctuations. Thanks to the canonical kinetic terms, the question amounts to whether or not there are any tachyonic modes.

5.1 The sphere case

Let us first describe what happens in the well-known sphere case. To this end, it is illuminating to write the spectrum (4.44)–(4.47) in the following way:

$$M^2 = \frac{4}{r_0^2}\left[l(l+1) - \left(\frac{P}{2}\right)^2\right] \tag{5.1}$$

where, for $\psi = \psi_+$, $P = m\omega - (m - N)\bar{\omega}$, $l = k + |1 + P/2|$ and we have the following definition of k in the two cases $P > -2$ and $P \le -2$:

$P > -2$:

- For $m \le -1/\omega$
$$k = n - m\omega - 1 \ge 0 \tag{5.2}$$

- For $-1/\omega < m \le N + 1/\bar{\omega}$
$$k = n \ge 0 \tag{5.3}$$

- For $m > N + 1/\bar{\omega}$
$$k = n + (m - N)\bar{\omega} - 1 > 0 \tag{5.4}$$

$P \le -2$:

- For $m \le N + 1/\bar{\omega}$
$$k = n + 1 - (m - N)\bar{\omega} \ge 0 \tag{5.5}$$

- For $N + 1/\bar{\omega} < m \le -1/\omega$
$$k = n \ge 0 \tag{5.6}$$

- For $m > -1/\omega$
$$k = n + m\omega + 1 > 0 \tag{5.7}$$

From here it is easy to confirm that in the sphere limit ($\omega, \bar{\omega} \to 1$), the spectrum for ψ_+ reduces to the expected form:

$$\mathcal{R}^2 M^2 = l(l+1) - \left(\frac{N}{2}\right)^2 \qquad \text{multiplicity} = 2l + 1\,, \tag{5.8}$$

where $\mathcal{R} = r_0/2$ represents the radius of the sphere, $l = k + |1 + N/2|$ and k is an integer which assumes all possible non-negative values ($k = 0, 1, 2, \ldots$). Recall that the results for ψ_- are obtained by taking $m \to -m$ and $N \to -N$. By using this information it is easy to see that a necessary and sufficient condition for the absence of tachyons in the sphere case is simply [18]

$$\left|N^I\right| \le 1 \quad \text{for every } I, \tag{5.9}$$

where we have restored the Lie algebra index I. To derive Inequality (5.9) one can use the fact that, in the sphere case, $k = 0$ is an allowed[18] value of k for every m, as can be checked by means of the definitions (5.2)–(5.7).

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[18]This property does not hold always for the conical-GGP solutions.

In order to satisfy (5.9) we must have that all the charges e^I corresponding to our sector (3.9) assume just one value, up to their sign. Moreover, we must have that the absolute value of all the charges in the hypermatter sector, $|e^i|$, is not smaller than $|e^I|$. These are consequences of the Dirac quantization condition (2.11). If we embed the background monopole in an Abelian factor of $\mathcal{G}$, these conditions are obviously satisfied as $e^I = 0$, for all I, in this case. However, for an embedding in a non-Abelian factor of $\mathcal{G}$, these conditions select only one possibility amongst the anomaly free models presented in table 1: the $E_7 \times E_6 \times \mathrm{U}(1)_R$ model, with the monopole embedded in E_6 under which all hypermultiplets are singlets [17–19].[19]

5.2 The conical-GGP case

Our purpose is now to see whether or not Condition (5.9) is valid also for the conical-GGP solutions. To this end one can analyze directly the explicit expressions for the spectrum given in (4.44)–(4.47) and simply study the inequality $M^2 \geq 0$ for those four expressions, which are valid in four different ranges of m. After a long but straightforward computation, the following results emerge.[20] A sufficient condition for the absence of tachyons in the conical-GGP solution is Constraint (5.9). Thus the warping and brane defects do not introduce new instabilities. Meanwhile, when both the tensions are non-negative ($T \geq 0$ and $\overline{T} \geq 0$), Constraint (5.9) is also a necessary condition for the absence of tachyons. In this case we have exactly the same situation as in the sphere case, so positive tension branes have no effect on stability at all. However, when at least one tension is negative ($T < 0$ and/or $\overline{T} < 0$), we can relax that constraint if the absolute value of a negative tension is large enough. The latter statement can be proved by analyzing the following special cases.

(i) $T < 0$ and $\overline{T} = 0$, that is $\omega < 1$ and $\overline{\omega} = 1$. This case corresponds to a solution with just one conical defect and a non-trivial warping. Here a necessary and sufficient condition for the absence of tachyons is

$$\left| N^I \right| \leq 1 + \frac{1}{3\omega} \quad \text{for every } I. \tag{5.10}$$

(ii) $T = \overline{T} < 0$, that is $\omega = \overline{\omega} < 1$. This set up corresponds to the unwarped rugby ball compactification with negative deficit angles. In this case a sufficient condition for the absence of tachyons is

$$\left| N^I \right| \leq \frac{4}{3\omega} \quad \text{for every } I. \tag{5.11}$$

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[19]There are other non-trivial examples. For instance, amongst the numerous models given in [20], if we again take the monopole to lie in a non-Abelian factor, stability selects the $\mathrm{SU}(2) \times \mathrm{U}(1)_R$ model with hyperino representation as follows: seven hyperinos transforming as a **3** of $\mathrm{SU}(2)$, two as a **5** and thirty-one as a **7** [25].

[20]Recall that we have not imposed the orbifold boundary conditions on the present spectrum. By eliminating some of the modes the orbifolding could relax some of the stability conditions. The conditions we state are, however, certainly sufficient also in this case.

Some details of the derivation of (5.10) and (5.11) are provided in appendix B. Since ω appears in the denominator of (5.10) and (5.11), it is clear that we can render stable an arbitrary large value of $|N^I|$ by choosing a small enough value of ω, that is by introducing a large negative tension brane. For example, if we want to stabilize the value $|N^I| = 2$, which is unstable in the sphere case, we have to choose a deficit angle $\delta \leq -4\pi$ in case (5.10), and $\delta \leq -\pi$ in case (5.11).

The main conclusion is that the conical-GGP solution is a stable solution for all 6D gauged supergravities, if we allow arbitrarily negative brane tensions. However, if we require that there are only non-negative tensions, the stability of the system exactly selects the same models as in the sphere case (with the additional topological constraint that the monopole cannot be embedded in $U(1)_R$ — see the end of subsection 2.2).

We conclude this section by explaining how to impose the orbifold conditions on the spectrum found above. Indeed, as we have already discussed, the orbifold could be necessary to avoid instabilities in the presence of negative tension branes. We first observe that we started from the coordinate system r, φ to analyse (4.1), as commented below (4.3). The fields V_r and V_φ are even under the orbifold action because, in this coordinate system, this action is $r \rightarrow r$ and $\varphi \rightarrow \varphi + \pi$ and so $dr \rightarrow dr$ and $d\varphi \rightarrow d\varphi$. Therefore, imposing the orbifold on the spectrum consists of removing the odd modes, that is the modes with m odd. As we have anticipated, the orbifold only projects out some of the modes that we found[21] and so Conditions (5.10) and (5.11) remain sufficient condition for the absence of tachyons. The main conclusion of the paper is therefore unchanged in the presence of the orbifold.

6. Conclusions

We have studied the stability of axi-symmetric brane world compactifications (conical-GGP solutions) in anomaly free, chiral, gauged 6D supergravity. Anomaly freedom is a central principle of quantum physics, and in six dimensional supergravity it places restrictions on the possible matter supermultiplets that can be present. Indeed, this lack of arbitrariness can be considered as one of the theory's most attractive features. Meanwhile, we remark that the central results of our work also apply to more general situations. For example, they are relevant for any 6D gauged supergravity (anomalous or otherwise) which has non-Abelian gauge groups, and also for the Yang-Mills extensions to the non-supersymmetric models of [29].

We began by considering the various types of bulk and brane scalar fluctuations that are present in the model, and in particular how they decouple at the bilinear level. We chose to project out the brane bending modes by placing the branes at orbifold fixed points. Then, the Salam-Sezgin sector (i.e. those scalars fluctuations descending from the supergravity-tensor multiplet and the $U(1)$ gauge multiplet in the direction of the background monopole) was considered for the axially-symmetric perturbations in [13], and no instabilities were found.

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[21]This implies that the hamiltonian of the effective Schroedinger problem is hermitian even if we perform the orbifolding.

The remaining sectors are then the hyperscalar fluctuations, and those scalar fluctuations of the gauge fields which are orthogonal to the monopole background in the Lie algebra of $\mathcal{G}$. The hyperscalar fluctuations can immediately be seen to have positive squared-masses in the 4D effective theory. Therefore, the main efforts in the present article were directed towards the scalar fluctuations of the said gauge fields, the sector which normally harbours instabilities.

We used the light-cone gauge to derive the bilinear action for these fluctuations [38]. In this way we obtained the linearized equations of motion, and found them to be a pair of coupled, second-order ODEs. We transformed them into a pair of Schroedinger equations, and found that it is possible to decouple them. The problem then reduces to the same form as that treated in [15], and we analytically derived the full spectra. The exact results that we obtained enabled us to draw both expected and surprising results, with regards to the stability of the compactifications.

As was observed long ago for the sphere-monopole compactification of these theories [24], in general we find a tachyonic instability in the scalar fluctuations of the gauge fields that are charged under the monopole background. In the case of the sphere, the necessary and sufficient condition for the absence of tachyons can be written as $|N^I| \leq 1$, where N^I are the integer monopole numbers carried by each gauge field [18]. If we embed the monopole background in an Abelian factor of the gauge group, then the compactification is stable. However, if the monopole happens to lie in a non-Abelian factor of the gauge group, then generically the compactification is unstable. For example, amongst the anomaly free models described in table 1, only one fulfills the necessary condition: the $E_7 \times E_6 \times U(1)_R$ model with the monopole embedded in the hidden E_6 [17–19].

We find that the same condition holds for the conical-GGP solutions that contain positive tension branes only. It is also a sufficient condition for models which incorporate negative tension branes. In other words, a GGP compactification is stable if its sphere limit is stable. Furthermore, with positive tension branes only, a GGP compactification is unstable if its sphere limit is unstable. However, it becomes possible to relax the constraint by incorporating large, negative tensions. This seems remarkable, given that negative tension branes are usually associated with instabilities rather than stability.

Meanwhile, the simplest way to obtain a stable conical-GGP solution appears to be to embed the monopole in an Abelian factor of the gauge group. Notice that, at least for the most elegant anomaly free models such as those in table 1, there is typically only one Abelian gauge factor available, which corresponds to the $U(1)_R$ gauged R-symmetry. It is known, however, from the Dirac quantization condition, that embedding the monopole in the $U(1)_R$ again leads to the requirement of at least one negative tension brane [1, 3]!

We should note, though, that there is actually a host of anomaly free models with extra drone $U(1)$'s [19, 20]. For a stable model with only positive tension branes, therefore, we must turn to one of these or to one of the more "miracolous" models discussed at the end of Subsection 5.1.

For these reasons, we have also discussed in detail the geometry induced by negative tension branes. The associated negative deficit angles give rise to so-called *saddle-cones*. As noted, the orbifolding projects out the brane-embedding fields, which are usually a source of

instability for negative tension branes. We mention here that, furthermore, the orbifolding gives rise to a chiral spectrum for the bulk fermion zero modes. Whilst models with positive tension branes give rise to a chiral spectrum even without the orbifold projection, those which include negative tension branes in general include zero modes of both chirality. This can be observed by considering the spectrum found in [15]. In addition we note that the main conclusion of ref. [15] — that the Kaluza-Klein mass scale and that of the internal volume can decouple in the presence of conical defects — also holds for the present mass spectrum in the case of negative tension branes.

In summary, we find that the conical-GGP solutions with positive tension branes are stable only for very limited classes of anomaly free theories and monopole embeddings. Such models, therefore, as used for example in the Supersymmetric Large Extra Dimension Scenario, are almost unique in character. Meanwhile, somewhat surprisingly, negative tension branes seem to allow for stability in a much wider class of models. It would certainly be interesting to obtain some physical intuition as to how the negative tension branes render unstable sphere compactifications stable.

Finally, given that the generic model is unstable, the big question is: where is the instability taking it to? Here we comment that since the tachyonic mass has its origin in the internal part of the 6D gauge kinetic term, which is semi-positive definite, we expect it to be stabilized at the quartic level. Moreover, the tachyonic masses are found in non-axially symmetric modes, so we might ask: is there a stable non-axially symmetric brane world solution to 6D gauged supergravity?

Acknowledgments

A. Bilinear action in the light-cone gauge

In order to derive the bilinear action for the scalar fluctuations of the gauge field orthogonal to the monopole background, we use the results of [38]. In that reference, a formalism was developed to analyze the spectrum of small perturbations about arbitrary solutions of Einstein, Yang-Mills and scalar systems, using the light-cone gauge.[22] For a warped background solution, with their scalars inactive, the model turns out to be identical to ours, up to the latter's presence of the dilaton.[23] In this appendix, we show how our model can in fact be transformed into exactly the system treated in [38].

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[22]Some discussions of the light cone gauge in field theory are given in [40, 41, 38].

[23]Our model also contains the Kalb-Ramond field and hyperscalars, but again, for the background and fluctuations of interest, these sectors does not contribute.

The bilinear action for the sector of interest about the background (2.3) will have contributions only from the gauge kinetic term:

$$S = -\frac{1}{4} \int d^6 X \sqrt{-G} \left[e^{\kappa\sigma/2} G^{MN} G^{PQ} Tr \left(F_{MP} F_{NQ} \right) \right] \tag{A.1}$$

We can make a conformal transformation to absorb the dilaton by defining $\hat{G}_{MN} = e^{\phi} G_{MN}$, where we recall that $\phi \equiv \kappa\sigma/2$. In the new frame, the action is identical to that considered in [38]:

$$S = -\frac{1}{4} \int d^6 X \sqrt{-\hat{G}} \left[\hat{G}^{MN} \hat{G}^{PQ} Tr \left(F_{MP} F_{NQ} \right) \right] \tag{A.2}$$

The conformal transformation, however, implies that our background metric becomes:

$$ds^2 = e^{A+\phi} \eta_{\mu\nu} dx^\mu dx^\nu + e^\phi d\rho^2 + e^{B+\phi} d\varphi^2 , \tag{A.3}$$

which differs from that considered in [38], where $ds^2 = e^A \eta_{\mu\nu} dx^\mu dx^\nu + d\rho^2 + e^B d\varphi^2$. We therefore make a coordinate tranformation $e^{\phi/2} d\rho \equiv dr$, so that our metric can be written as:

$$ds^2 = e^{\hat{A}} \eta_{\mu\nu} dx^\mu dx^\nu + dr^2 + e^{\hat{B}} d\varphi^2 , \tag{A.4}$$

with $\hat{A} \equiv A + \phi$ and $\hat{B} \equiv B + \phi$. The action (A.2) of course remains invariant under the change of coordinates.

After these tricks, we can follow exactly the same steps performed in [38] to remove the gauge degrees of freedom and obtain the dynamics of the physical fields. We expand to bilinear order, transform to light-cone coordinates, fix to the light-cone gauge and eliminate the redundant degrees of freedom using their equations of motion. The final result, for the spin-0 fields V_r and V_φ orthogonal to the background monopole, is then the following bilinear action:

$$S_2(V, V) = -\frac{1}{2} \int d^6 X \sqrt{-\hat{G}} \, Tr \left[\partial_\mu V_i \partial^\mu V^i + D_i V_j D^i V^j - 2(\partial_r \hat{A})^2 V_r^2 \right.$$
$$\left. -2(\partial_r \hat{A}) V_r D_i V^i + \hat{R}_{ij} V^i V^j + 2\overline{g} \, F_{ij} V^i \times V^j \right] , \tag{A.5}$$

where now $\hat{G}_{MN}$, σ and F_{ij} refer to the background fields and V_i to the fluctuations. The index i runs over r, φ and all indices are raised and lowered with $\hat{G}_{MN}$.

B. Details of stability analysis

In this appendix we give some intermediate steps to obtain the results of Subsection 5.2, concerning the stability analysis for the conical-GGP solutions in the presence of negative tension branes. Indeed, only in the presence of at least one negative tension brane our results differ from the stability constraint given in (5.9) that is valid in the sphere case. Therefore, as in Subsection 5.2, here we focus on the following special cases

(i) $T < 0$ and $\overline{T} = 0$, that is $\omega < 1$ and $\overline{\omega} = 1$,

(ii) $T = \overline{T} < 0$, that is $\omega = \overline{\omega} < 1$,

whose analysis is enough to obtain the main results of the present paper. More precisely, in the following we show how to obtain Constraints (5.10) and (5.11).

Let us first consider Case (i). The mass squared spectrum for the scalar sector (3.9), which we are interested in, is given in Equations (4.44)–(4.47), for the ψ_+ wave function,[24] and one has to take into account all of them to perform a complete analysis. However, here we only consider Equation (4.45) because the analysis of (4.44), (4.46) and (4.47) is analogous. By using Ansatz (i), Equation (4.45) becomes

$$\lambda = \left(n + \frac{3}{2}\right)^2 - \frac{1}{4} + \left(n + \frac{3}{2}\right)[m\omega - (m - N)], \tag{B.1}$$

where $\lambda \equiv r_0^2 M^2/4$. We recall that (4.45) is valid for $-1/\omega < m \leq N + 1/\bar{\omega}$, which, for $\bar{\omega} = 1$, becomes

$$m\omega > -1 \quad \text{and} \quad m - N \leq 1. \tag{B.2}$$

Now we observe that

$$\lambda = n^2 + [3 + m\omega - (m - N)]\, n + 2 + \frac{3}{2}[m\omega - (m - N)]$$
$$\geq 2 + \frac{3}{2}[m\omega - (m - N)] = \lambda_0, \tag{B.3}$$

where λ_0 is λ evaluated at $n = 0$ and we used $3 + m\omega - (m - N) > 1$, which is a consequence of (B.2). Therefore, we have $\lambda \geq 0$ if $\lambda_0 \geq 0$, namely if

$$m\omega \geq m - N - \frac{4}{3}. \tag{B.4}$$

Now we observe that, for $m - N \leq 0$, Constraint (B.4) is always satisfied because of (B.2). Therefore, tachyons can only be present in the case[25] $m = N + 1$ and by plugging this value of m into (B.4) we obtain

$$N \geq -1 - \frac{1}{3\omega}. \tag{B.5}$$

The corresponding constraint for ψ_- can be obtained by transforming $N \to -N$ in (B.5); this leads to $N \leq 1 + 1/(3\omega)$, which, together with (B.5), gives exactly (5.10). By using a similar method, we further checked that (5.10) is a necessary and sufficient condition for $\lambda \geq 0$ for the full spectrum including also (4.44), (4.46) and (4.47).

Let us now consider Case (ii). Again we present only the analysis of Equation (4.45), which, for $\omega = \bar{\omega}$, is

$$\lambda = \left(n + \frac{3}{2}\right)^2 - \frac{1}{4} + \left(n + \frac{3}{2}\right)N\omega. \tag{B.6}$$

The mass squared given in (4.45) is valid for $-1/\omega < m \leq N + 1/\bar{\omega}$, which reduces to

$$m\omega > -1 \quad \text{and} \quad (m - N)\omega \leq 1 \tag{B.7}$$

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[24]We recall that the spectrum for ψ_- can be obtained by transforming $m \to -m$ and $N \to -N$.

[25]We observe that Constraints (B.2) forbid $m > N + 1$.

in the case $\omega = \overline{\omega}$. As we did in Case (i), we observe that

$$\lambda = n^2 + (3 + N\omega)\, n + 2 + \frac{3}{2}N\omega \;\geq\; 2 + \frac{3}{2}N\omega = \lambda_0, \tag{B.8}$$

where we used $3 + N\omega > 1$, which is a consequence of (B.7). So a sufficient condition for $\lambda \geq 0$ is

$$N \geq -\frac{4}{3\omega} \tag{B.9}$$

and, by also taking into account the corresponding constraint for ψ_- ($N \leq 4/(3\omega)$), we obtain (5.11). Analogously we checked that (5.11) is a sufficient condition for $\lambda \geq 0$ for (4.44), (4.46) and (4.47) as well.

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